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Unknown input and state estimation of nonlinear systems using a multiple model approach

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Abstract—**This paper presents a new recursive filter to joint input and state estimation for noisy discrete time Takagi-Sugeno (T-S) fuzzy models. For each local linear model one local filter is designed using Kalman filter theory. Steady state and unknown input solutions can be found for each of the local filters. The global filter is a linear combination of linear filters. The local filter is time invariant, which greatly reduces the computational complexity of the global filter. The global filter is optimal in the sense of the unbiased minimum variance (UMV) criteria.**

Index Terms—**TS fuzzy model, State and unknown input estimation, nonlinear dynamic systems, minimum-variance.**

I. INTRODUCTION

The simultaneous estimation of the state and the unknown inputs (UI) of a system has received much research attention ever since the original works[1, 2, 3, 4, 6, 17, 18, 19, 20, 21] and is a key problem in many engineering applications due to the practical and/or economical problems arising when measuring signals of a process.

A robust state estimation with respect to UI plays therefore a fundamental role in numerous system control and/or supervision strategies. With regard to this last purpose, an UI can generally be employed in order to modelling an actuator failure and/or an abnormal behaviour of an internal component of the system.

Clearly, the state and the UI estimations can be employed for providing fault symptoms of the systems in order to make the system more reliable and safe.

Nonlinear filtering problems arise in many practical applications, e.g., financial estimation, biological and industrial processes, target localization and tracking, robots and robotic manipulators, and traffic state estimation. As is well known, a general approach to solve these problems is generalizing the Kalman filter paradigm for nonlinear systems, e.g., the extended Kalman filter (EKF). It is noted that the EKF is a first-order filter that propagates only the mean and covariance of the filtering densities, which however may diverge or provide poor state estimates due to its inherent first-order Taylor approximation of the nonlinear model. Other efforts to improve on the EKF have been developed, e.g., the DD2 filter [7], the unscented Kalman filter (UKF) [8], the derivative-free version of the EKF [9], the Gaussian particle filter (GPF) [10], the cubature Kalman filters (CKF) [11], and the derivativefree estimation method [12]. As addressed in [9], all the above filters attempt to improve the EKF by representing state uncertainty with a different ensemble set of state vectors. To the best of the author's knowledge, all the above-mentioned nonlinear estimator design methods are not yet applied to UIF problem for nonlinear stochastic systems. A heuristic approach of applying the aforementioned filters to solve the UIF problem of nonlinear stochastic systems is to augment the system state with the unknown inputs, and then apply the dedicated filtering method to the obtained augmented system. Notice that in this approach the unknown input model is always needed and assumed beforehand; as shown in the previous works [13]-[15], this approach may not perform well for arbitrary unknown inputs. A possible solution to remedy this problem is to propose ERTSF-like recursive algorithms that can optimally estimate the system state in light of arbitrary unknown input values [13]-[15]. However, it should be stressed that all these unknown-input decoupled nonlinear estimators (UIDNEs) are derived based on a direct application and extension of the ERTSF [5], which in general may only yield a specific linear combination of the unknown input vector.

In other words, only the estimable unknown input estimates from the measured outputs are provided and the remaining unestimable unknown input estimates are ignored. On the other hand, few research results concern simultaneous state and input estimation for nonlinear systems [16], which only considered linear measurement and limited system nonlinearity. Thus, the state estimation problem of applying the UIF method for general nonlinear stochastic systems still remains open.

In this paper, we extend the previous works [5] and continue the research line in investigating the applications of the UIF method to solve the addressed state estimation problem of nonlinear stochastic systems with unknown inputs.

The paper is organized as follows. In Section 2, the statement of the problem is addressed and the problems encountered. In Section 3, the fuzzy Kalman filter is presented. The proposed filter is presented in section 4. Illustrations of applying the proposed framework through an example is given in Section 4 to show the usefulness of the proposed results. Finally, conclusions are highlighted in the last section.

II. PROBLEM FORMULATION

Nonlinear systems can be approximated as locally linear systems in much the same way non-linear function can be approximated as piecewise linear function. Nonlinear systems can be represented by fuzzy linear models of the following form

If $z_1(k)$ is F_{i1} and $\cdots$ $z_g(k)$ is F_{ig} then

$$x(k+1)=A_ix(k)+B_iu(k)+G_id(k)+w(k) \quad (1)$$

$$y(k)=C_ix(k)+H_id(k)+v(k),\ \left((i=1,...,L)\right) \quad (2)$$

This is referred to as a Takagi-Sugeno (T-S) fuzzy model. The z_j are premise variables, k is the time index, F_{ij} are fuzzy sets, $x(k)\in\Re^n$ is the state vector, $u(k)\Re^p$ is the deterministic input, $d(k)\Re^m$ is the unknown input vector and $y(k)\in\Re^r$ is the measurement vector. The process noise $w(k)\in\Re^n$ and the measurement noise $v(k)\in\Re^p$ are assumed to be mutually uncorrelated zeros-mean white random signals with nonsingular covariance matrices $Q_k=\left(w_k w_k^T\right)\geq 0$ and $R_k=E\left(v_k v_k^T\right)>0$. Each of the local models of (1) and (2) is a linear time invariant model. The fuzzy combination of these local models results in the global model:

$$x(k+1)=\sum_{i=1}^{L}h_i\left(z(k)\right)\left(A_ix(k)+B_iu(k)+G_id(k)+w(k)\right) \quad (3)$$

$$y(k)=\sum_{i=1}^{L}h_i\left(z(k)\right)\left(C_ix(k)+H_id(k)+v(k)\right) \quad (4)$$

Where $h_i\left(z(k)\right)$ the membership grades are defined as:

$$h_i\left(z(k)\right)=\frac{\mu_i\left(z(k)\right)}{\mu(k)} \quad (5)$$

$$\mu_i\left(z(k)\right)=\prod_{j=1}^{g}F_{ij}=\left(z_j(k)\right) \quad (6)$$

$$\mu(k)=\sum_{i=1}^{L}\mu_i\left(z(k)\right) \quad (7)$$

$$z(k)=\left[z_1(k),\ ...,\ z_g(k)\right] \quad (8)$$

$F_{ij}\left(z_j(k)\right)$ is the membership grade of $z_j(k)$ in F_{ij}. Note that $h_i\left(z(k)\right)\in[0,1]$.From (3) and (5) we can see that:

$$\sum_{i=1}^{L}h_i\left(z(k)\right)=1 \quad (9)$$

From (2) we can derive:

$$x(k+1)=A(k)x(k)+B(k)u(k)+G(k)d(k)+w(k) \quad (10)$$

$$y(k)=C(k)x(k)+H(k)d(k)+v(k) \quad (11)$$

where $A(k)$, $B(k)$, $G(k)$, $C(k)$ and $H(k)$ are given as :

$$A(k)=\sum_{i=1}^{L}h_i\left(z(k)\right)A_i,\quad B(k)=\sum_{i=1}^{L}h_i\left(z(k)\right)B_i,$$

$$G(k)=\sum_{i=1}^{L}h_i\left(z(k)\right)G_i,\quad C(k)=\sum_{i=1}^{L}h_i\left(z(k)\right)C_i$$

$$H(k)=\sum_{i=1}^{L}h_i\left(z(k)\right)H_i \quad (12)$$

The global model is a fuzzy combination of L Local linear time - invariant models, can be represented as a time -varying model. If the premise variable $z(k)$ are function of the state or control, then the model is nonlinear . however, if the premise variable are independent of the state and control, then the model is linear. Now we define L discrete time signals $x_i(k)$, $d_i(k)$ and $y_i(k)$ as :

$$x_i(k)=h_i\left(z(k)\right)x(k),\qquad d_i(k)=h_i\left(z(k)\right)d(k)$$

$$y_i(k)=h_i\left(z(k)\right)y(k),\qquad x(k)=\sum_{i=1}^{L}x_i(k) \quad (13)$$

$$d(k)=\sum_{i=1}^{L}d_i(k),\qquad y(k)=\sum_{i=1}^{L}y_i(k) \quad (14)$$

The dynamic of the $x_i(k)$ and $y_i(k)$ signals is presented in the following form:

$$x_i(k+1) = A_i x_i(k) + G_i d_i(k) + h_i(z(k)) B_i u(k) + h_i(z(k)) w(k) \tag{15}$$

$$y_i(k) = C_i x_i(k) + H_i d_i(k) + h_i(z(k)) v(k) \tag{16}$$

III. FUZZY KALMAN FILTER

In this section we present a Kalman filter for each local systems given by in the following form

$$x(k+1) = \sum_{i=1}^{L} h_i(z(k))(A_i x(k) + B_i u(k) + w(k)) \tag{17}$$

$$y(k) = \sum_{i=1}^{L} h_i(z(k))[C_i x(k) + v(k)] \tag{18}$$

For a nonlinear dynamic systems that is described by the T-S fuzzy model, a FKF can be designed to estimate the systems state vector .

A local linear filter can be designed for each local linear dynamic model using Kalman theory. At an operating point, the local filter is associated with each fuzzy rule as given below

If $z_1(k)$ is F_{i1} and $\cdots$ $z_g(k)$ is F_{ig} then

$$\hat{x}_i(k/k) = \hat{x}_i(k/k-1) + K_i(k)(y_i(k) - C_i \hat{x}_i(k/k-1)) \tag{19}$$

$$\hat{x}_i(k+1/k) = A_i \hat{x}_i(k/k) + h_i(z(k)) B_i u(k) \tag{20}$$

The Kalman gain matrix $K_i(k)$ in equation (19) can be calculated from the following set of equations:

$$\tilde{R}_i^{-1}(k) = \left(C_i P_i(k/k) C_i^T + R_i\right)^{-1} \tag{21}$$

$$K_i(k) = P_i(k/k) C_i^T \tilde{R}_i^{-1}(k) \tag{22}$$

$$P_i(k/k) = (I - K(k) C_i) P_i(k/k-1) \tag{23}$$

$$P_i(k+1/k) = A_i P_i(k/k) A_i^T - A_i K_i(k) C_i P_i(k/k) A_i^T + Q_i \tag{24}$$

where $P_i(k/k-1)$ and $P_i(k/k)$ are the covariance matrices of errors in predicted and update state estimates of the ith local filter, respectively. The overall state estimation is a nonlinear combination of individual local filter outputs. The overal filter dynamics will then be a weighted sum of individual linear filters, given by

$$\hat{x}(k/k) = \sum_{i=1}^{L} \hat{x}_i(k+1/k) \tag{25}$$

$$\hat{x}(k+1/k) = \sum_{i=1}^{L} h_i(z(k))(A_i \hat{x}_i(k/k) + B_i u(k)) \tag{26}$$

IV. STATE AND UNKNOWN INPUT ESTIMATION

Consider the systems given in the following form:

$$x(k+1) = \sum_{i=1}^{L} h_i(z(k))(A_i x(k) + B_i u(k) + G_i d(k) + w(k)) \tag{27}$$

$$y(k) = \sum_{i=1}^{L} h_i(z(k))(C_i x(k) + H_i d(k) + v(k)) \tag{28}$$

This section deals with UIF estimation problem, based on the coupled multiple model (27) and (28), using the ERTSF (5) . The filter given in the three step:

- Estimation of the unknown input,
- The measurement update
- Time update

These three steps are given by:

- Estimation of the unknown input,

$$\tilde{R}_i(k) = \left(C_i P_i^x(k/k-1) C_i^T + R_i\right) \tag{29}$$

$$\hat{d}_i(k) = M_i(k)(y_i(k) - C_i \hat{x}_i(k/k-1)) \tag{30}$$

$$M_i(k) = \left(H_i \tilde{R}_i^{-1}(k) H_i^T\right)^{-1} H_i^T \tilde{R}_i^{-1}(k) \tag{31}$$

$$P_i^d(k) = \left(H_i^T R_k^{-1}(k) H_i\right)^{-1} \tag{32}$$

- Measurement update

$$K_i(k) = P_i^x(k/k-1) C_i^T \tilde{R}_i^{-1}(k) \tag{33}$$

$$\hat{x}_i(k/k) = \hat{x}_i(k/k-1) + K_i(k)(y_i(k) - C_i \hat{x}_i(k/k-1)) - H_i \hat{d}_i(k) \tag{34}$$

$$P_i^x(k/k) = P_i(k/k-1) - K_i(k) \times \left(\tilde{R}_i(k) - H_i P_i^d(k) H_i^T\right) K_i^T(k) \tag{35}$$

- Time update

$$\hat{x}_i(k+1/k) = A_i \hat{x}_i(k/k) + h_i(z(k)) B_i u(k) + h_i(z(k)) G_i \hat{d}_i(k) \tag{36}$$

$$P_i^x(k+1/k) = [A_i \quad G_i] \begin{bmatrix} P_i^x(k/k) & P_i^{xd}(k) \\ P_i^{dx}(k) & P_i^d(k) \end{bmatrix} \begin{bmatrix} A_i^T \\ G_i^T \end{bmatrix} + Q_k \quad (37)$$

The overal filter dynamics will then be a weighted sum of individual linear filters, given by

$$\hat{x}(k+1/k) = \sum_{i=1}^{L} \hat{x}_i(k+1/k) \quad (38)$$

$$\hat{x}(k+1/k) = \sum_{i=1}^{L} h_i(z(k))\left(A_i\hat{x}_i(k/k) + B_i u(k) + G_i\hat{d}(k)\right) \quad (39)$$

V. Simulation Example

To show the proposed result, we consider the following for submodels of a coupled multiple mode where the parameters are given by:

$$A = \begin{bmatrix} -\frac{144.034}{v_{ref}} & \frac{58.896}{v_{ref}^2} - 1 \\ 29.859 & -\frac{170.981}{v_{ref}} \end{bmatrix}, \; B = \begin{bmatrix} \frac{52.802}{v_{ref}} \\ 40.939 \end{bmatrix}$$

$$C = \begin{bmatrix} -152.756 & \frac{62.463}{v_{ref}^2} \\ 0 & 1 \end{bmatrix}, \; D = \begin{bmatrix} 56 \\ 0 \end{bmatrix}$$

In order to obtain the TS fuzzy model it is necessary to define two premise variables. These are

$$z_1(t) = \frac{1}{v_{ref}}, \qquad z_2(t) = \frac{1}{v_{ref}^2}$$

Now, the matrices A, B and C can be defined in the following form:

$$A(z) = \begin{bmatrix} -144.034z_1 & 58.896z_2 - 1 \\ 29.859 & -170.981z_1 \end{bmatrix}, \; B(z) = \begin{bmatrix} 52.802z_1 \\ 40.939 \end{bmatrix},$$

$$C(z) = \begin{bmatrix} -152.756 & 62.463z_1 \\ 0 & 1 \end{bmatrix}$$

The calculation of the minimum and maximum values of $z_1(t)$ and $z_2(t)$ for $v_{ref} = [5 \quad 55]$ m/s are:

$$\max z_1 = z_1^+ = 0.2 \qquad \max z_2 = z_2^+ = 0.04$$

$$\min z_1 = z_1^- = 0.018 \qquad \min z_2 = z_2^- = 3.305e^{-4}$$

From the maximum and minimum values of $z_1(t)$ and $z_2(t)$ the membership function are calculated as follows

$$F_{11}(z_1^+) = \frac{z_1 - z_1^-}{z_1^+ - z_1^-} \qquad F_{12}(z_1^-) = \frac{z_1^+ - z_1}{z_1^+ - z_1^-}$$

$$F_{21}(z_2^+) = \frac{z_2 - z_2^-}{z_2^+ - z_2^-} \qquad F_{22}(z_2^-) = \frac{z_2^+ - z_2}{z_2^+ - z_2^-}$$

After the discretization of each subsystem, using $10\,ms$ as simple time, the vehicle dynamic model is given by the deffuzzifiation as:

$$x(k+1) = \sum_{i=1}^{4} h_i(z(k))\left(A_i x(k) + B_i u(k) + w(k)\right)$$

$$y(k) = \sum_{i=1}^{4} h_i(z(k))\left(C_i x(k) + v(k)\right)$$

where

$$h_1(z(k)) = F_{11}(z(k)) \times F_{21}(z(k))$$

$$h_2(z(k)) = F_{11}(z(k)) \times F_{22}(z(k))$$

$$h_3(z(k)) = F_{12}(z(k)) \times F_{21}(z(k))$$

$$h_4(z(k)) = F_{12}(z(k)) \times F_{22}(z(k))$$

$$A_1 = \begin{bmatrix} 0.7512 & 0.0099 \\ 0.2181 & 0.7118 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.7486 & -0.0072 \\ 0.2178 & 0.7093 \end{bmatrix} \; A_3 = \begin{bmatrix} 0.9761 & 0.0132 \\ 0.2904 & 0.9714 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0.9727 & -0.0095 \\ 0.2900 & 0.9680 \end{bmatrix} \; B_1 = \begin{bmatrix} 0.0941 \\ 0.3598 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.0901 \\ 0.3594 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.0122 \\ 0.4048 \end{bmatrix}, \; B_4 = \begin{bmatrix} 0.0075 \\ 0.4043 \end{bmatrix},$$

$$D = \begin{bmatrix} 56 \\ 0 \end{bmatrix} \; C_1 = C_2 = \begin{bmatrix} -152.7568 & 12.4926 \\ 0 & 1 \end{bmatrix}, \; ,$$

$$C_3 = C_4 = \begin{bmatrix} -152.7568 & 1.1357 \\ 0 & 1 \end{bmatrix}$$

$$R=\begin{bmatrix}0.5 & 0\\ 0 & 0.1\end{bmatrix},\ Q=\begin{bmatrix}0.9 & 0\\ 0 & 0.1\end{bmatrix}$$

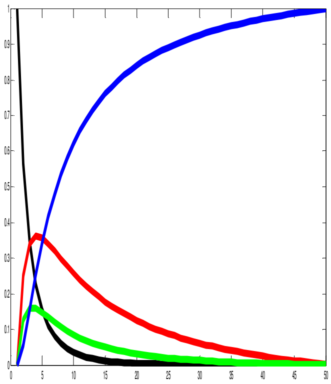

Figure 1: Weighting functions

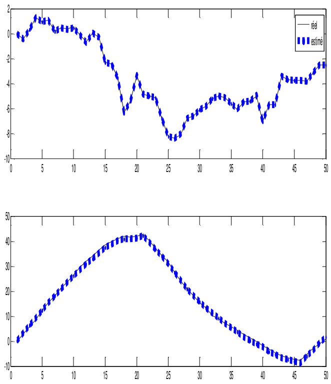

Figure2: State and its estimate

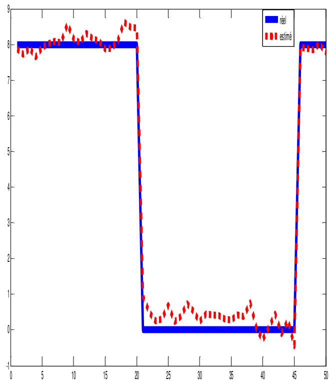

Figure 5. Unknown input and its estimate

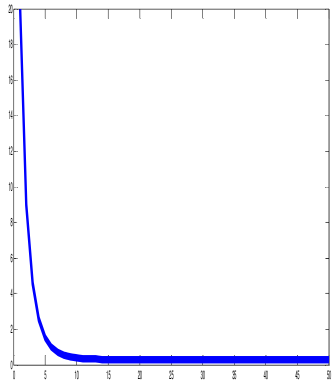

Figure 3. Trace of covariance P^x

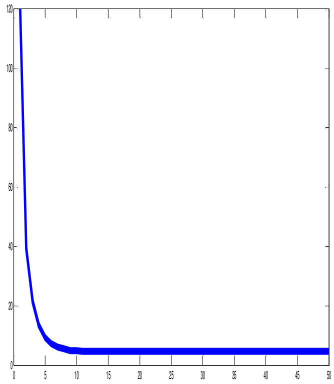

Figure4 : Trace of covariance P^d

Table 1: Evaluation of the RMSE values

Rmse x_1	**Rmsef$_1$**	**Rmse x_2**	***TraceP^x***	***Trace P^d***
0.2398	***0.1798***	***0.0024***	***4.6188***	***0.3002***

VI. . CONCLUSION

In this present paper, an extension of ERTSF is presented for estimating the state variables and the unknown inputs of nonlinear stochastic systems modelled by a coupled multiple model. The suggested filter can be used, as an extension of the classic Kalman filter scheme, in the detection and the isolation of sensor and actuator failures.

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